

Scale-Independent Calculation of $\sin^2 \theta_{eff}^{lept}$.

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Abstract

We present a calculation of the electroweak mixing parameter $\sin^2 \theta_{eff}^{lept}$ that incorporates known higher order effects, shares the desirable convergence properties of the \overline{MS} scheme, and has the important theoretical advantage of being strictly independent of the electroweak scale in finite orders of perturbation theory. We also show how this formulation can be extended to the calculation of the W mass M_W . The results provide accurate, scale-independent evaluations of these important parameters, as functions of the Higgs boson mass M_H , and are compared with previous calculations in order to analyze the scheme and scale dependence of the electroweak corrections.

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The accurate calculation of the electroweak mixing parameter $\sin^2 \theta_{eff}^{lept}$ and the W -boson mass M_W rank among the most important objectives of precision studies of the Standard Model (SM). In fact, these parameters have been measured very accurately and place important constraints on the Higgs boson mass M_H .

Some time ago it was shown that the incorporation of the $\mathcal{O}(\alpha^2 M_t^2/M_W^2)$ contributions [1] greatly reduces the scheme and scale dependence of the radiative corrections, as well as the upper bound on M_H [2, 3]. In these papers the calculations of $\sin^2 \theta_{eff}^{lept}$ and M_W were carried out in two implementations of the on-shell scheme of renormalization [4, 5], denoted as OSI and OSII, as well as in the \overline{MS} framework [6, 7, 8, 9, 10]. Comparison among the three calculations led then to an analysis of the scheme dependence of the electroweak corrections and, by inference, to an estimate of the theoretical error arising from the truncation of the perturbative series. The on-shell and \overline{MS} formulations, which are the most frequently employed in electroweak calculations, present a number of relative advantages and disadvantages. The on-shell approach is very “physical”, in the sense that it employs renormalized parameters, such as α , G_μ , and M_W , that are physical observables and, therefore, scale independent. The \overline{MS} calculations follow closely the structure of the unrenormalized theory and, for this reason, avoid the emergence of large corrections that are frequently induced by renormalization. Thus, they have very desirable convergence properties. On the other hand, they employ parameters, such as $\hat{\alpha}(\mu)$ and $\sin^2 \hat{\theta}_W(\mu)$, which are inherently scale dependent. Since $\sin^2 \theta_{eff}^{lept}$ and M_W are observable quantities, their evaluation, if carried out to all orders in perturbation theory, should lead to scale-independent results. Practical calculations, however, involve a truncation of the perturbative series and this induces a residual scale dependence. For instance, among the three schemes discussed in Refs. [2, 3], only OSII is scale independent.

The aim of the present paper is to present a calculation of $\sin^2 \theta_{eff}^{lept}$ that shares the desirable convergence properties of the \overline{MS} scheme, but has the important theoretical advantage of being strictly independent of the electroweak scale in finite orders of perturbation theory. We also show how this formalism can be extended to the calculation of M_W and applied to the analysis of the scheme dependence of the electroweak corrections.

Our starting point is based on two basic relations of the \overline{MS} renormali-

zation scheme, as applied to electroweak physics [6, 8, 11]:

$$\hat{s}^2 \hat{c}^2 = \frac{A^2}{M_Z^2 (1 - \Delta\hat{r})} , \quad (1)$$

$$s_{eff}^2 = \left[1 + \frac{\hat{e}^2}{\hat{s}^2} \Delta\hat{k}(M_Z^2) \right] \hat{s}^2 , \quad (2)$$

where $A^2 = \pi\alpha/\sqrt{2}G_\mu$, $\hat{s}^2 \equiv 1 - \hat{c}^2$ and s_{eff}^2 are abbreviations for the \overline{MS} parameter $\sin^2 \hat{\theta}_W(\mu)$ and $\sin^2 \theta_{eff}^{lept}$, respectively, and $\Delta\hat{r}$ and $(\hat{e}^2/\hat{s}^2) \Delta\hat{k}(M_Z^2)$ are the relevant electroweak corrections. As s_{eff}^2 is defined in terms of the $Z_0 \rightarrow \bar{l}l$ couplings on resonance, $\Delta\hat{k}$ is evaluated at $q^2 = M_Z^2$. Numerically, $(\hat{e}^2/\hat{s}^2) \Delta\hat{k}(M_Z^2)$ is very small, of $\mathcal{O}(4 \times 10^{-4})$ for $\mu \approx M_Z$. It is important to note that $\hat{s}^2, \hat{e}^2, \Delta\hat{r}$ and $\Delta\hat{k}(M_Z^2)$ are scale-dependent quantities, while α, G_μ, M_Z , and s_{eff}^2 , being physical observables, are not.

Since the present knowledge of the irreducible two-loop corrections to $\Delta\hat{r}$ and $(\hat{e}^2/\hat{s}^2) \Delta\hat{k}(M_Z^2)$ is restricted to contributions enhanced by powers $(M_t^2/M_Z^2)^n$ ($n = 1, 2$), our strategy to obtain scale-independent expressions is to combine Eq. (1) with Eq. (2) retaining only such contributions (as well as complete one loop effects), and expressing the results in terms of scale-independent couplings.

Recalling that at the one-loop level $\Delta\hat{k}(M_Z^2)$ depends only logarithmically on M_t/M_Z [1, 11] and combining Eq. (1) with Eq. (2), we find

$$s_{eff}^2 c_{eff}^2 = \frac{A^2}{M_Z^2 (1 - \Delta\hat{r}) \left[1 - \frac{\hat{e}^2}{\hat{s}^2} \Delta\hat{k} \left(1 - \frac{s_{eff}^2}{c_{eff}^2} \right) \right]} . \quad (3)$$

At the one-loop level [2, 6, 8]

$$1 - \Delta\hat{r} = 1 + \frac{2\delta e}{e} \Big|_{\overline{MS}} + \frac{e^2}{\hat{s}^2} \Delta\hat{\rho} + \dots ,$$

where $\frac{\delta e}{e} \Big|_{\overline{MS}}$ is the charge renormalization counterterm in the \overline{MS} scheme, $\Delta\hat{\rho}$ is given in Eq. (4) of Ref. [2], and the dots stand for one-loop contributions not involving the factor M_t^2/M_Z^2 . Noting that $(1 + \frac{2\delta e}{e} \Big|_{\overline{MS}}) \hat{e}^2 = e^2$ [8] and $\hat{e}^2/\hat{s}^2 = G_\mu 8M_W^2/\sqrt{2} + \dots$ [2, 8], where the dots indicate higher order terms, we see that Eq. (3) can be written in the form

$$s_{eff}^2 c_{eff}^2 = \frac{A^2}{M_Z^2 (1 - \Delta r_{eff})} , \quad (4)$$

$$\Delta r_{eff} = \Delta \hat{r} + \frac{e^2}{s_{eff}^2} \Delta \hat{k} \left(1 - \frac{s_{eff}^2}{c_{eff}^2} \right) (1 + x_t) , \quad (5)$$

where

$$x_t = \frac{3G_\mu}{8\sqrt{2}\pi^2} M_t^2 \quad (6)$$

is the leading one-loop contribution to $(\hat{e}^2/\hat{s}^2) \Delta \hat{\rho}$ and we have neglected two-loop corrections not proportional to $(M_t^2/M_Z^2)^n$. The one-loop approximation to Eqs. (4, 5) has been recently applied to discuss the mass scale of new physics in the Higgs-less scenario [12] and the evidence for electroweak bosonic corrections in the SM [13].

At this stage, it is convenient to express $\Delta \hat{r}$ in terms of the corrections $\Delta \hat{r}_W$, $\Delta \hat{\rho}$, and \hat{f} discussed in Refs. [2, 7, 8]. Using Eqs. (15b, 15a, 8b, 8a) of Ref. [8], we obtain

$$\Delta \hat{r} = \Delta \hat{r}_W - (e^2/s_{eff}^2) \Delta \hat{\rho} - (e^2/s_{eff}^2) x_t \left[2\Delta \hat{\rho} - (\Delta \hat{\rho})_{lead} - \hat{f} + \Delta \hat{k} \right] , \quad (7)$$

where we have again neglected two-loop contributions not proportional to $(M_t^2/M_Z^2)^n$ and $(\Delta \hat{\rho})_{lead} = (3/64\pi^2) M_t^2/M_W^2$. Combining Eq. (5) and Eq. (7), we have

$$\begin{aligned} \Delta r_{eff} &= \Delta \hat{r}_W - \frac{e^2}{s_{eff}^2} \left[\Delta \hat{\rho} - \Delta \hat{k} \left(1 - \frac{s_{eff}^2}{c_{eff}^2} \right) \right] \\ &\quad - \frac{e^2}{s_{eff}^2} x_t \left[2\Delta \hat{\rho} - (\Delta \hat{\rho})_{lead} - \hat{f} + \Delta \hat{k} \frac{s_{eff}^2}{c_{eff}^2} \right] . \end{aligned} \quad (8)$$

The corrections $\Delta \hat{r}_W$, $(e^2/s_{eff}^2) \Delta \hat{\rho}$ and $(e^2/s_{eff}^2) \Delta \hat{k}$ include irreducible two-loop contributions proportional to $(M_t^2/M_Z^2)^n$ [1, 2]. In Eq. (8) \hat{s}^2 has been substituted everywhere by the scale-independent parameter s_{eff}^2 . However, $\Delta \hat{r}_W$, $\Delta \hat{\rho}$, $\Delta \hat{k}$, and \hat{f} depend also on $c^2 = M_W^2/M_Z^2$, where c^2 is an abbreviation for the on-shell parameter $\cos^2 \theta_W$ [4, 5]. In order to obtain an expression

that depends only on c_{eff}^2 , we proceed as follows: i) M_W^2 is replaced everywhere by $c^2 M_Z^2$, ii) in all the contributions that are explicitly of two-loop order we substitute $c^2 \rightarrow c_{eff}^2$, since the difference is of third order, and iii) in the one-loop terms we perform a Taylor expansion, exemplified by

$$\Delta\hat{\rho}(c_{eff}^2, c^2) = \Delta\hat{\rho}(c_{eff}^2, c_{eff}^2) + \left. \frac{\partial \Delta\hat{\rho}}{\partial c^2} \right|_{c^2=c_{eff}^2} (c^2 - c_{eff}^2) + \dots \quad (9)$$

In the leading contribution to $\Delta\hat{\rho}$, proportional to M_t^2 , $c^2 - c_{eff}^2$ in Eq. (9) is replaced by the complete one-loop expression (see Eq. (14))

$$c^2 - c_{eff}^2 = \left(G_\mu M_Z^2 / \sqrt{2} \right) 8 c_{eff}^4 \left[\Delta\hat{\rho} + (s_{eff}^2 / c_{eff}^2) \Delta\hat{k} \right], \quad (10)$$

while in the terms not proportional to M_t^2 (and this includes $\Delta\hat{r}_W$ and $\Delta\hat{k}$) we employ only the leading part $c^2 - c_{eff}^2 = c_{eff}^2 x_t$. In this way, the calculation of s_{eff}^2 is completely decoupled from that of M_W , and can be carried out iteratively on the basis of Eq. (4) and Eq. (8).

In order to evaluate M_W , it is convenient to consider the relation [8]

$$c^2 = \hat{\rho} \hat{c}^2 = (1 - \hat{s}^2) \left[1 - (\hat{e}^2 / \hat{s}^2) \Delta\hat{\rho} \right]^{-1}. \quad (11)$$

Expressing \hat{s}^2 in the first factor in terms of s_{eff}^2 via Eq. (2), and neglecting again two-loop effects not proportional to $(M_t^2 / M_Z^2)^n$, we obtain

$$c^2 = c_{eff}^2 \left\{ 1 - \frac{\hat{e}^2}{\hat{s}^2} \left[\Delta\hat{\rho} + \frac{s_{eff}^2}{c_{eff}^2} \Delta\hat{k} \left(1 - \frac{\hat{e}^2}{\hat{s}^2} \Delta\hat{\rho} \right) \right] \right\}^{-1}. \quad (12)$$

Next, we insert the relation

$$\hat{e}^2 / \hat{s}^2 = \left(G_\mu / \sqrt{2} \right) 8 M_W^2 \left[1 - (\hat{e}^2 / \hat{s}^2) \hat{f} \right], \quad (13)$$

which follows from Eq.(8) of Ref. [2]. This leads to

$$M_W^2 / M_Z^2 = c_{eff}^2 \left\{ 1 - \frac{G_\mu}{\sqrt{2}} 8 M_W^2 \left[\Delta\hat{\rho} + \frac{s_{eff}^2}{c_{eff}^2} \Delta\hat{k} (1 - x_t) - \hat{f} x_t \right] \right\}^{-1}. \quad (14)$$

Since the only one-loop contribution proportional to M_t^2 is $(\Delta\hat{\rho})_{lead}$, and this is independent of \hat{c}^2 , this parameter can be replaced by c_{eff}^2 everywhere in

the expression between curly brackets. Replacing again $M_W^2 = c^2 M_Z^2$, this expression can be regarded as a function $G(c_{eff}^2, c^2)$. Performing a Taylor expansion about $c^2 = c_{eff}^2$, analogous to Eq. (9), we obtain then a function that depends only on c_{eff}^2 . This fact insures the strict electroweak-scale independence of the result and permits the numerical evaluation of M_W on the basis of Eq. (14) and the c_{eff}^2 values obtained before.

Since s_{eff}^2 plays the role of the basic renormalized electroweak mixing parameter, for brevity we will refer to the present framework as the “effective” (EFF) scheme of renormalization. Numerical results for M_W and s_{eff}^2 based on this scheme are presented in Tables 1 and 2, as functions of M_H , using $G_\mu = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$, $M_Z = 91.1875 \text{ GeV}$, $\alpha_s(M_Z) = 0.119$, and $M_t = 174.3 \text{ GeV}$ for the pole mass of the top quark. Table 1 employs the traditional value $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$ [14] for the five-flavor hadronic contribution to $\alpha(M_Z)$, while Table 2 is based on $\Delta\alpha_h^{(5)} = 0.02770 \pm 0.00017$ [15], one of the recent “theory driven” calculations. In both Tables, QCD contributions are implemented in the $\mu_t = \overline{M}_t(\mu_t)$ scheme explained, for instance, in Ref. [2] ($\overline{M}_t(\mu_t)$ is the \overline{MS} running top quark mass at scale μ). As the dependence of $\Delta\hat{\rho}(c_{eff}^2, c^2)$, $\Delta\hat{r}_W(c_{eff}^2, c^2)$, $\Delta\hat{k}(c_{eff}^2, c^2)$, $\hat{f}(c_{eff}^2, c^2)$, and $G(c_{eff}^2, c^2)$ on c^2 is rather involved, we have found convenient to evaluate numerically, rather than analytically, the derivatives with respect to c^2 exemplified in Eq. (9).

Calculations in the \overline{MS} -scheme traditionally employ $\mu = M_Z$ when evaluating electroweak corrections at or near the Z^0 -resonance region [2]. Detailed comparisons to six significant figures show that the differences (\overline{MS} at $\mu = M_Z$ minus EFF calculations) are $\delta s_{eff}^2 = (1.6, 1.4, 0.9, 0.6, 0.5) \times 10^{-5}$ and $\delta M_W = (0.2, 0.3, 0.4, 0.2, -0.1) \text{ MeV}$ for $M_H = (65, 100, 300, 600, 1000) \text{ GeV}$, respectively, when $\Delta\alpha_h^{(5)} = 0.02804$ is employed, with essentially the same values for $\Delta\alpha_h^{(5)} = 0.02770$. We have also compared the results of the \overline{MS} and EFF frameworks when the M_t implementation of the QCD corrections is used [2]. In this case the differences are $\delta s_{eff}^2 = (1.2, 1.0, 0.6, 0.5, 0.4) \times 10^{-5}$ and $\delta M_W = (0.1, 0.1, 0.0, -0.3, -0.7) \text{ MeV}$ for $\Delta\alpha_h^{(5)} = 0.02804$, and essentially the same for $\Delta\alpha_h^{(5)} = 0.02770$. Thus, the differences between the \overline{MS} ($\mu = M_Z$) and EFF calculations are very small, with maximal variations of $\delta s_{eff}^2 = 1.6 \times 10^{-5}$ (at $M_H = 65 \text{ GeV}$ with μ_t -QCD corrections), and $\delta M_W = -0.7 \text{ MeV}$ (at $M_H = 1 \text{ TeV}$ with M_t -QCD evaluations).

Figs. 1 and 2 compare the \overline{MS} and the EFF calculations of s_{eff}^2 and

M_W , as functions of the scale μ , for $M_H = 100 \text{ GeV}$, $\Delta\alpha_h^{(5)} = 0.02804$, and μ_t -QCD corrections. We see that the difference δs_{eff}^2 is within $\pm 1 \times 10^{-5}$ in the range $26 \text{ GeV} \leq \mu \leq 202 \text{ GeV}$, but becomes negative and sizable for small and large values of μ . A very similar pattern holds for the difference δM_W . Thus the EFF calculations give support to the \overline{MS} results of Refs. [2, 3], which employ $\mu = M_Z$, and at the same time remove the inherent ambiguities associated with the choice of the electroweak scale. The effect of such ambiguities on the analysis of the scheme-dependence is discussed later on. It is also interesting to note that, although physicists usually choose \overline{MS} scales on the basis of energies characteristic of the physical observables under consideration, there are important cases in which scales obtained by optimization methods (BLM [16], FAC [17], PMS [18]) are very different. Recent examples include the relation between pole and \overline{MS} pole masses, where optimization methods [19] led to extraordinarily accurate predictions of third-order coefficients [20], and the QED corrections to μ -decay [21].

A scale-independent estimate of renormalization-scheme differences can be achieved by comparing the present results with the OSII calculations, which are based on the on-shell framework and are also scale independent. For $\Delta\alpha_h^{(5)} = 0.02804$, μ_t -QCD corrections and the same inputs, the differences (OSII minus EFF calculations) are $\delta s_{eff}^2 = (3.0, 3.3, 3.6, 3.2, 2.3) \times 10^{-5}$ and $\delta M_W = -(1.4, 1.4, 1.4, 1.4, 1.2) \text{ MeV}$, for $M_H = (65, 100, 300, 600, 1000) \text{ GeV}$. If, instead, we employ the M_t -QCD implementation, we obtain $\delta s_{eff}^2 = (4.2, 4.3, 3.9, 3.1, 1.7) \times 10^{-5}$ and $\delta M_W = -(1.9, 1.8, 1.7, 1.5, 1.1) \text{ MeV}$. Comparing the results, for given M_H , of the four calculations (OSII and EFF results, with μ_t or M_t -QCD corrections), we find maximal variations of $\approx 5 \times 10^{-5}$ in s_{eff}^2 and $\approx 3 \text{ MeV}$ in M_W . These conclusions are very similar to those reported in Ref. [2]. For $\Delta\alpha_h^{(5)} = 0.02770$, the maximal variations are nearly identical to those given above. As pointed out in [3], the QCD uncertainties are expected to be larger than indicated by the difference between the μ_t and M_t approaches, reaching $\pm 3 \times 10^{-5}$ in s_{eff}^2 and $\pm 5 \text{ MeV}$ in M_W . Thus, for $M_H = 100 \text{ GeV}$ the overall estimated uncertainty is $\approx 6 \times 10^{-5}$ in s_{eff}^2 and $\approx 7 \text{ MeV}$ in M_W . Using the approximate relations $\delta M_H/M_H \approx 1.9 \times 10^3 \delta s_{eff}^2$ and $\delta M_H/M_H \approx -1.6 \times 10^{-2} \delta M_W/\text{MeV}$, which can be gleaned from Ref. [3], we see that the theoretical uncertainties induced by such scheme dependences amount to $\delta M_H/M_H \approx \pm 0.11$ in both the s_{eff}^2 and M_W cases.

It is important to note that, since the s_{eff}^2 results of OSII are larger than

those of \overline{MS} for arbitrary electroweak scale, there is no μ value for which the two calculations coincide and, in fact, their difference increases for small and large μ values (cf. Fig. 1). In contrast, in the M_W case one can choose μ such that the OSII and \overline{MS} calculations agree exactly. For instance, for $M_H = 100 \text{ GeV}$, this occurs at $\mu \approx 45 \text{ GeV}$ and $\mu \approx 240 \text{ GeV}$. This shows that the residual scale ambiguity of the \overline{MS} calculations complicates the analysis of scheme-dependence and that it is, in fact, highly advantageous to compare scale-independent calculations, as we have done above.

One may also compare the M_W calculation in the EFF scheme with a recent and more complete on-shell analysis that incorporates all the two-loop contributions to Δr that contain a fermion loop [22]. For equal inputs, we find $\delta M_W = (6.1, 5.3, 2.7, 1.6, 0.7) \text{ MeV}$, ($\delta M_W \equiv \text{EFF minus Ref. [22], using } M_t\text{-QCD corrections}$), with the effective calculation leading to slightly larger M_H values for given M_W . It is not clear, however, that this comparison is a good test of scheme dependence, since Ref. [22] includes a class of two-loop effects not contained in the current EFF calculation of M_W .

As a final application, we list the values of M_H and its upper-bounds obtained with the EFF calculations on the basis of the current experimental values of s_{eff}^2 and M_W , without taking into account the experimental lower bounds obtained by direct searches. Using the world average values $s_{eff}^2 = 0.23146 \pm 0.00017$, $M_t = (174.3 \pm 5.1) \text{ GeV}$, $\hat{\alpha}_s(M_Z) = 0.119 \pm 0.002$ [23], and μ_t -QCD corrections, we find $M_H = (86_{-41}^{+78} \text{ GeV})$, $M_H^{95} = 248 \text{ GeV}$ for $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$, and $M_H = (109_{-41}^{+66} \text{ GeV})$, $M_H^{95} = 238 \text{ GeV}$ for $\Delta\alpha_h^{(5)} = 0.02770 \pm 0.00016$ (M_H^{95} is the 95% CL upper bound). Employing $M_W = (80.436 \pm 0.037) \text{ GeV}$ [23] we find $M_H = (28_{-28}^{+60} \text{ GeV})$, $M_H^{95} = 153 \text{ GeV}$ for $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$, and $M_H = (33_{-33}^{+62} \text{ GeV})$, $M_H^{95} = 162 \text{ GeV}$ for $\Delta\alpha_h^{(5)} = 0.02770 \pm 0.00016$. We recall that the range $M_H \lesssim 113 \text{ GeV}$ is already excluded by direct searches at the 95% CL. It is worth noting that among all the calculational schemes we have discussed ($\overline{MS}(\mu = M_Z)$, OSII, Ref.[22], and EFF), the latter gives the smallest (largest) value of $s_{eff}^2(M_W)$, for given M_H . Since $s_{eff}^2(M_W)$ is a monotonically increasing (decreasing) function of M_H , the EFF calculations lead to the largest values of M_H .

In summary, we have discussed and implemented a novel framework of renormalization in which s_{eff}^2 plays the role of the renormalized electroweak mixing parameter. This scheme shares the desirable convergence properties of the \overline{MS} approach, with the important theoretical advantage that the cal-

culations are strictly independent of the electroweak scale in finite orders of perturbation theory. Thus, it also shares the attractive properties of the on-shell scheme. When applied to the evaluation of the basic parameters s_{eff}^2 and M_W , it leads to results that are very close to those in the \overline{MS} scheme, provided that the scale in the latter calculation is chosen in the neighborhood of M_Z . Thus, it gives strong support to the \overline{MS} calculations carried out in the past and, at the same time, it removes the ambiguity associated with the choice of the electroweak scale. As stressed in the paper, the elimination of this dependence is important, not only in order to obtain unambiguous results, but also in the analysis of the scheme-dependence of the electroweak corrections.

The authors are greatly indebted to G. Degrassi and P. Gambino for detailed discussions and access to their computational codes. This research was supported by NSF grant PHY-0070787.

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Table 1: Predicted values of M_W and s_{eff}^2 in the EFF renormalization scheme. QCD corrections based on μ_t -parametrization. $M_t = 174.3 \text{ GeV}$, $\hat{\alpha}_s(M_Z^2) = 0.119$, $\Delta\alpha_{had}^{(5)} = 0.02804$

$M_H [\text{GeV}]$	$M_w [\text{GeV}]$	$\sin^2 \theta_{eff}^{lept}$
65	80.401	0.23132
100	80.378	0.23153
300	80.305	0.23211
600	80.251	0.23250
1000	80.212	0.23278

Table 2: As in Table 1, but with $\Delta\alpha_{had}^{(5)} = 0.02770$.

$M_H [\text{GeV}]$	$M_w [\text{GeV}]$	$\sin^2 \theta_{eff}^{lept}$
65	80.407	0.23120
100	80.384	0.23142
300	80.310	0.23199
600	80.257	0.23238
1000	80.218	0.23266

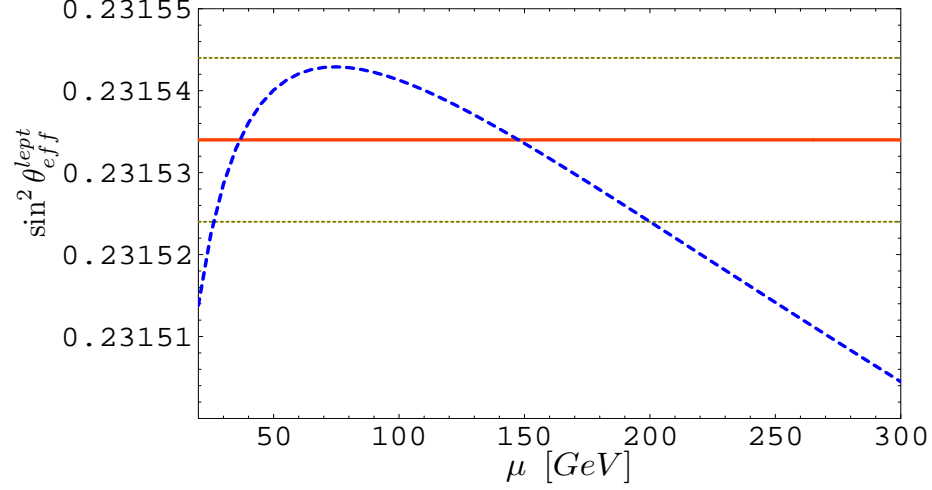


Figure 1: Scale dependence of s_{eff}^2 in the \overline{MS} (dashed line) and EFF (solid line) schemes for $M_H = 100 \text{ GeV}$ and the input parameters listed in Table 1. The light dotted lines define a range of $\pm 1 \times 10^{-5}$ around the EFF result.

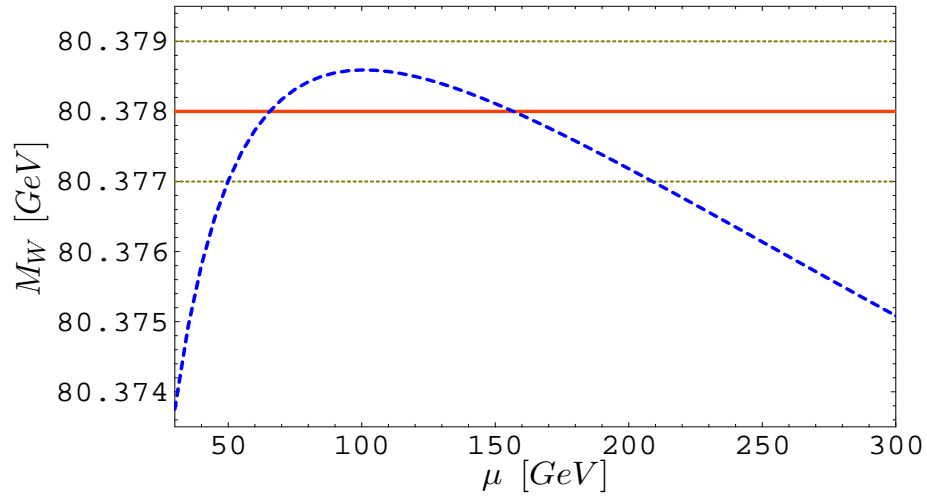


Figure 2: Scale dependence of M_W in the \overline{MS} (dashed line) and EFF (solid line) schemes for $M_H = 100 \text{ GeV}$ and the input parameters listed in Table 1. The light dotted lines define a range of $\pm 1 \text{ MeV}$ around the EFF result.